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- Blinder, Alan S. (1998). Central Banking in Theory and Practice. The MIT Press.

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1. Introduction

It is generally observed that central banks have a tendency to overstay their stance. Their monetary policy often remains tight for too long – causing recessions, and often remains easy for too long – allowing inflation pressure to build up. Alan Blinder (1998) mentions two main reasons why this may happen.

First, he observes that the central banks have (or had? back in 1998) not yet fully internalized the dynamic-programming way of thinking, that is deciding today on a policy path the central bank will take from today until time T, rather than just deciding what the central bank should do today and start thinking about tomorrow's decision only when tomorrow has arrived.

Second, Alan Blinder reckons that central banking by committee might favour sluggish changes in monetary policy stances. Blinder thinks that "had Newton served on more faculty committees at Cambridge, his first law of motion might have read: A decisionmaking body at rest or in motion tends to stay at rest or in motion in the same direction unless acted upon by an outside force." This inertia has both advantages and disadvantages: One advantage clearly is that no single person can control the monetary policy. Since broad agreement is required for consensus, decisions tend to regress towards the mean and are optimal in the long run, i.e. not too extreme in any direction away from the mean.

The major disadvantage clearly is that decisionmaking by committee may contribute to systematic monetary policy errors by maintaining a certain policy stance for too long. Nevertheless, it is sound to think that central banking by committee provides a net benefit to society.

2. Too little, too late

Equation Section 2

2.1 Introduction

The 2005 paper by Petra Gerlach-Kristen (PGK) is situated in this field of analyzing decisionmaking by committee and provides a nice application of the Kalman filter to a signal extraction problem. PGK uses the Kalman filter to model the formation of beliefs about the optimal interest rate by members of a central banking committee.
The main question to which the paper seeks an answer is whether a majority vote is preferable to a consensus procedure in order to reach a decision on the monetary policy. The European Central Bank (ECB) for example follows the consensus procedure and is often criticised to wait too long before raising or lowering the interest rate (by too little). A counterexample would be the Bank of England (BoE) where monetary policy decisions are taken by majority vote. PGK lets the policymakers take decisions by either majority vote or consensus and performs a simulation exercise in order to find out whether decision taking by consensus exacerbates the "too little, too late" problem.

2.2 The model

The decisions of the central bank are taken by a central banking committee consisting of n members. We would have $n=9$ for the BoE and $n=18$ for the ECB. PGK makes the assumption that policymakers agree on policy goals, do not behave strategically and are uncertain about the state of the economy.
Given the state of the economy at any time t, there is an optimal level of the interest rate i_t^*, which is not directly observable to the policymakers. The optimal interest rate follows the AR(1) process

$$i_t^* = \rho i_{t-1}^* + e_t \tag{2.1}$$

where $0<\rho<1$ and $e_t \sim N\left(0,\sigma_*^2\right)$.

The monetary policy instrument is the interest rate i_t – changeable by steps of size s – and its level is determined in the monetary policy committee by consensus or by majority vote. Every committee member has to form an optimal estimate of i_t^* – i.e. form his beliefs about optimal i_t – based on his noisy observation of i_t^* and on his noisy observation of the other committee members' optimal estimates which he makes in the meeting just before the vote.
Policymaker j's noisy observation of i_t^* is

$$i_{j,t} = i_t^* + u_{j,t} \tag{2.2}$$

with $u_{j,t} \sim N\left(0,\sigma_u^2\right)$ $\forall j$ and $Cov\left(u_{j,t},u_{k,t}\right)=\varphi$ for $j \neq k$.
Policymaker j's noisy observation of policymaker k's optimal estimate is

$$i_{jk,t} = i_{k,t} + w_{jk,t} \tag{2.3}$$

with $w_{jk,t} \sim N\left(0,\sigma_w^2\right)$. Policymaker j knows ρ, σ_*^2, σ_u^2, φ and σ_w^2 and observes $n-1$ $i_{jk,t}$'s.

The problem of extracting i_t^* can be cast in state-space form and the Kalman filter thus provides an optimal estimate $i_{t|t}^*$ of i_t^*.

Policymaker j's observation equation is

$$\mathbf{i}_{j,t} = \mathbf{Z} i_t^* + \boldsymbol{\varepsilon}_t \tag{2.4}$$

where $\mathbf{i}_{j,t} = \begin{bmatrix} i_{j1,t} & \ldots & i_{j(j-1),t} & i_{j,t} & i_{j(j+1),t} & \ldots & i_{jn,t} \end{bmatrix}'$ is an $n \times 1$ vector, $\mathbf{Z} = \begin{bmatrix} 1 & \ldots & 1 \end{bmatrix}'$ is a $n \times 1$ vecor and $\boldsymbol{\varepsilon}_t = \begin{bmatrix} u_t + w_t & \ldots & u_t & \ldots & u_t + w_t \end{bmatrix}'$ is the vector of disturbances where u_t sits in the j-th row and $Var(\varepsilon_t) = \mathbf{H} = \begin{bmatrix} \sigma_u^2 + \sigma_v^2 & \varphi & \cdots & \cdots & \varphi \\ \varphi & \ddots & & & \vdots \\ \vdots & & \sigma_u^2 & & \vdots \\ \vdots & & & \ddots & \varphi \\ \varphi & \cdots & \cdots & \varphi & \sigma_u^2 + \sigma_v^2 \end{bmatrix}$ where $h_{jj} = \sigma_u^2$.

The state equation is given by (2.1).

The prediction equations are

$$i_{t|t-1}^* = \rho i_{t-1|t-1}^* \tag{2.5}$$

and

$$P_{t|t-1} = \rho^2 P_{t-1|t-1} + \sigma_*^2 \tag{2.6}$$

The updating equations are

$$i_{t|t}^* = i_{t|t-1}^* + \mathbf{K}_t \left(\mathbf{i}_{j,t} - \mathbf{Z} i_{t|t-1}^* \right) \tag{2.7}$$

and

$$P_{t|t} = P_{t|t-1} - \mathbf{K}_t \mathbf{Z} P_{t|t-1} \tag{2.8}$$

where $\mathbf{K}_t = P_{t|t-1} \mathbf{Z}' \left(\mathbf{Z} P_{t|t-1} \mathbf{Z}' + \mathbf{H} \right)^{-1}$ is a $1 \times n$ vector. A more compact form of the Kalman filter is obtained by substituting (2.5) in (2.7) to get

$$\begin{aligned} i_{t|t}^* &= \rho i_{t-1|t-1}^* + \mathbf{K}_t \left(\mathbf{i}_{j,t} - \rho \mathbf{Z} i_{t-1|t-1}^* \right) \\ &= \mathbf{K}_t \mathbf{i}_{j,t} + \rho \left(1 - \mathbf{K}_t \mathbf{Z} \right) i_{t-1|t-1}^* \end{aligned} \tag{2.9}$$

and by substituting (2.8) in (2.6) to get

$$\begin{aligned} P_{t|t-1} &= \rho^2 \left[P_{t-1|t-2} - \mathbf{K}_t \mathbf{Z} P_{t-1|t-2} \right] + \sigma_*^2 \\ &= \rho^2 P_{t-1|t-2} - \rho^2 \mathbf{K}_t \mathbf{Z} P_{t-1|t-2} + \sigma_*^2 \end{aligned} \tag{2.10}$$

2.3 Intermezzo: The steady-state Kalman filter

The Kalman filter requires a lot of computational operations, which is why it is of interest to look at the convergence properties of the Kalman filter. If, for example, some updating equation would converge to a steady-state value, it needn't be computed at every iteration any more.

Consider class of state-space models where some of the system matrices, namely $\mathbf{Z}$, $\mathbf{H}$, $\mathbf{T}$ and $\mathbf{Q}$, are the time invariant

$$\mathbf{y}_t = \mathbf{Z}\boldsymbol{\alpha}_t + \mathbf{d}_t + \boldsymbol{\varepsilon}_t, \qquad t = 1,\ldots,T \tag{2.11}$$

$$\boldsymbol{\alpha}_t = \mathbf{T}\boldsymbol{\alpha}_{t-1} + \boldsymbol{\eta}_t, \qquad t = 1,\ldots,T \tag{2.12}$$

where $E[\boldsymbol{\varepsilon}_t] = \mathbf{0}$, $Var(\boldsymbol{\varepsilon}_t) = \mathbf{H}$, $E[\boldsymbol{\eta}_t] = \mathbf{0}$ and $Var(\boldsymbol{\eta}_t) = \mathbf{Q}$.

The state process is stationary if the characteristic roots of $\mathbf{T}$ lie inside the unit root circle. The Kalman filter applied to (2.11) and (2.12) is said to be in a steady-state if the error covariance matrix is time-invariant

$$\mathbf{P}_{t+1|t} = \overline{\mathbf{P}} \tag{2.13}$$

If this is the case, the updating equation for the state is

$$\begin{aligned}
\mathbf{a}_{t+1|t} &= \mathbf{T}\mathbf{a}_{t|t} + \mathbf{c}_t \\
&= \mathbf{T}\mathbf{a}_{t|t-1} + \mathbf{T}\overline{\mathbf{K}}\left(\mathbf{y}_t - \mathbf{Z}_t\mathbf{a}_{t|t-1} - \mathbf{d}_t\right) + \mathbf{c}_t \\
&= \left(\mathbf{T} - \mathbf{T}\overline{\mathbf{K}}\mathbf{Z}\right)\mathbf{a}_{t|t-1} + \mathbf{T}\overline{\mathbf{K}}\mathbf{y}_t + \left(\mathbf{c}_t - \mathbf{T}\overline{\mathbf{K}}\mathbf{d}_t\right)
\end{aligned} \tag{2.14}$$

where the steady-state Kalman gain is $\overline{\mathbf{K}} = \overline{\mathbf{P}}\mathbf{Z}'\left(\mathbf{Z}\overline{\mathbf{P}}\mathbf{Z}' + \mathbf{H}\right)^{-1}$.

The Kalman filter has a steady-state solution if there exists a $\overline{\mathbf{P}}$ which satisfies the Riccati equation (cf. expression (2.10))

$$\mathbf{P}_{t+1|t} = \mathbf{T}_{t+1}\left[\mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{Z}_t'\left(\mathbf{Z}_t\mathbf{P}_{t|t-1}\mathbf{Z}_t' + \mathbf{H}_t\right)^{-1}\mathbf{Z}_t\mathbf{P}_{t|t-1}\right]\mathbf{T}_{t+1}' + \mathbf{Q}_{t+1} \tag{2.15}$$

If there exists a solution to (2.15), then we get

$$\mathbf{P}_{t+1|t} = \mathbf{P}_{t|t-1} = \overline{\mathbf{P}} \tag{2.16}$$

and obtain the algebraic Riccati equation (ARE)

$$\overline{\mathbf{P}} - \mathbf{T}\overline{\mathbf{P}}\mathbf{T}' + \mathbf{T}\overline{\mathbf{P}}\mathbf{Z}'\left(\mathbf{Z}\overline{\mathbf{P}}\mathbf{Z}' + \mathbf{H}\right)^{-1}\mathbf{Z}\overline{\mathbf{P}}\mathbf{T}' - \mathbf{Q} = \mathbf{0} \tag{2.17}$$

The conditions for this to happen are: If the eigenvalues of the $m \times m$ matrix $\mathbf{T}$ lie inside the unit root circle and if the matrices $Var(\boldsymbol{\varepsilon}_t) = \mathbf{H}$ and $Var(\boldsymbol{\eta}_t) = \mathbf{Q}$ are positive semidefinite symmetric with either $\mathbf{H}$ or $\mathbf{Q}$ strictly positive definite, then the sequence of the MSE matrices $\left\{\mathbf{P}_{t|t-1}\right\}_{t=2}^{T}$ converges to a unique positive semidefinite steady-state matrix $\overline{\mathbf{P}}$ that satisfies the Riccati equation, irrespective of the positive semidefinite symmetric starting value chosen for $\mathbf{P}_{1|0}$.

2.4 Solving for the steady-state

Since the policymaker's signal extraction problem has by assumption the properties $0 < \rho < 1$, $\sigma_*^2 > 0$ and strictly positive definite $\mathbf{H}$, the steady-state Kalman filter is given by

$$i_{t|t}^* = \bar{\mathbf{K}}\mathbf{i}_{j,t} + \rho\left(1 - \bar{\mathbf{K}}\mathbf{Z}\right)i_{t-1|t-1}^* \tag{2.18}$$

and

$$\bar{P} = \rho^2\bar{P} - \rho^2\bar{\mathbf{K}}\mathbf{Z}\bar{P} + \sigma_*^2 \tag{2.19}$$

where $\bar{\mathbf{K}} = \bar{P}\mathbf{Z}'\left(\mathbf{Z}\bar{P}\mathbf{Z}' + \mathbf{H}\right)^{-1}$ (cf. expressions (4) to (6) in PGK).

At the end of the monetary policy committee meeting every policymaker has extracted his optimal estimate $i_{t|t}^*$ and he will vote for the target rate i_t closest to his estimate. PGK assumes that i_t is changed if a certain degree of consensus m is reached.
Note that the bigger the uncertainty is, the smaller $\mathbf{K}$ turns out to be, which implies slower adjustment of $i_{t|t}^*$ in function of the observation $\mathbf{i}_{j,t}$. Further, note that it can be shown that $Var\left(i_{t|t}\right) < Var\left(i_t^*\right)$ and that therefore the reactions of the policymaker tend to be too small.

2.5 Simulations

PGK simulates the model in order to determine how the choice of m in the interval from 50% to 100% influences the interest rate setting. She calibrates the model with $s = 25$ basis points, $n = 11$, $\rho = 0.95$, and $\sigma_*^2 = \sigma_u^2 = \sigma_w^2 = 0.1$. The covariance of measurement errors φ can take values of 0, 0.05 or 0.09.
PGK measures the optimality of the monetary policy by

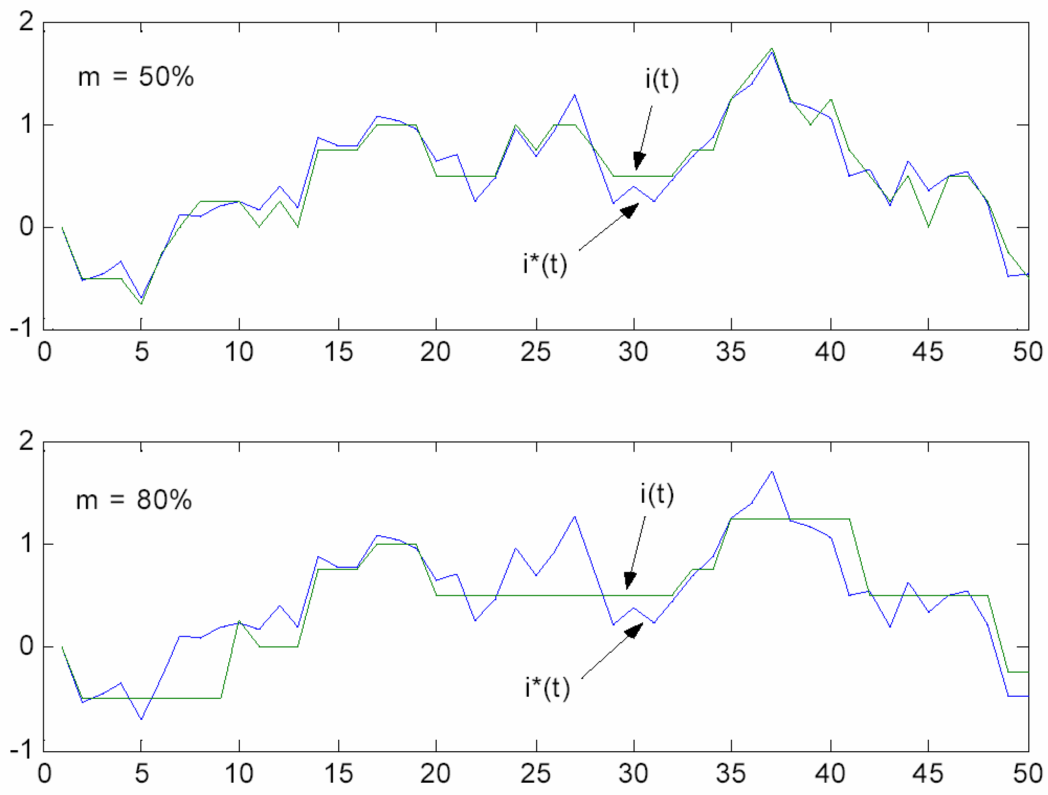
$$deviation = \frac{1}{T}\sum_{t=1}^{T}\left|i_t - i_t^*\right| \tag{2.20}$$

and the sluggishness of the monetary policy by

$$lag = \min_l\left\{\frac{1}{T-l}\sum_{t=1+l}^{T}\left|i_t - i_{t-l}^*\right|\right\} \tag{2.21}$$

where T is the number of draws in a simulation. The bigger the lag, the „too late" the monetary policy and the bigger the deviation, the „too little" the adjustment of i_t.

Figure 1 shows clearly that the difference between the optimal and the instrument rate is bigger for $m = 80\%$ than in the $m = 50\%$ case. Since the $i_{t|t}^*$'s of very single member lag behind i_t^* by construction, it is but logical that it takes more time for a larger majority to form, thus if m is bigger, lag will be longer and $deviation$ bigger.



Note: Simulations for $n = 11$, $s = 0.25$, $\rho = 0.95$, $\sigma_*^2 = \sigma_u^2 = \sigma_w^2 = 0.1$, $\varphi = 0$.

Figure 1: Simulated interest rate paths; source PGK 2005.

Table 1 shows *deviation* and *lag* in function of different values for the parameters m and φ. First, note that *deviation* and *lag* always increase in function of m. Second, note that *deviation* is largest for $\varphi = 0.09$ if m is small, but largest for $\varphi = 0$ if m is big. An intuitive explanation is that when $\varphi = 0.09$ policymakers have similar views and reach an agreement faster, whereas for $\varphi = 0$ the other policymakers' observations are more informative and they get a better $i^*_{t|t}$, which is good if m is small.

	$\varphi = 0$		$\varphi = 0.05$		$\varphi = 0.09$	
	deviation	*lag*	*deviation*	*lag*	*deviation*	*lag*
$m =$						
50%	9.8	0	16.7	0	20.0	0
60%	12.1	0	18.2	0	20.8	0
70%	15.2	0	19.9	0	22.0	0
80%	20.4	0	22.2	0	23.5	0
90%	30.7	1	27.1	1	26.0	1
100%	56.7	4	38.4	2	31.3	1

Note: *deviation* in bps, *lag* in periods. Simulations for $T = 10,000$, $n = 11$, $s = 0.25$, $\rho = 0.95$, $\sigma_*^2 = \sigma_u^2 = \sigma_w^2 = 0.1$.

Table 1: The cost of consensus; source PGK 2005.

2.6 Conclusion

Uncertainty about the optimal level of i_t causes monetary policy to be adjusted too late and by too little. This phenomenon tends to be exacerbated when the decision is formed by consensus rather than majority vote.