Empirical Macroeconomics
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Recent Developments in SVAR

1 Introduction[1]

On the methodical side, the first two lectures are intended to familiarize you with the method of vector autoregressions (VAR). While the first lecture has introduced basic concepts (VAR, SVAR), this second lecture presents more recent methodical developments. First, we discuss an extension of the VAR, the factor augmented VAR (FAVAR). In short, a FAVAR tries to exploit richer information sets without giving up the statistical advantages of restricting the analysis to a small number of time series. Second, we briefly discuss the Lucas critique and the Lucas programme. We will see that under a stable monetary policy rule, the VAR approach can be directly used to identify and assess the effects of monetary policy shocks. However, if the policy rule is instable VARs will be rather employed to derive stylized facts about the response of the economy to monetary policy shocks. These stylized facts can then be used to evaluate structural macroeconomic models that forecast the effects of monetary policy under changing monetary policy rules and institutions. This application of VARs lies at the heart of the so called Lucas-programme. Third, the concept of modest policy changes as suggested by Leeper and Zah (2003) is introduced. Leeper and Zah (2003) present formal criteria to evaluate the importance of expcetational effects and thus to evaluate the applicability of reduced form models.

On the macroeconomic side, the lecture still is about the macroeconomic effects of monetary policy shocks.

2 The FAVAR Approach

Source: Bernanke et al. (2005)

2.1 Introduction

The traditional VAR and SVAR approach to identify monetary policy shocks has been subject to various criticisms, in particular concerning the relatively small amount of information considered by a VAR. A standard VAR rarely contains more than 6 variables, since the loss of degrees of freedom – with regard to rather small estimation samples - seriously complicates statistical inference. At the same time it is questionable whether the central bank only considers the information set that is covered by the VAR.

Bernanke et al. (2005, 288) highlight several problems related to the above finding: First, if the central bank has more information to decide on its policy than considered by the VAR, the measure of monetary policy shocks will typically be contaminated. An example is the so-called "price-puzzle", which is the finding in the VAR literature that a contractionary monetary policy shock is followed by an increase in prices rather than a decline. If however the FED systematically tightens policy in anticipation of future inflation and if signals of future inflation are not adequately captured by the VAR, then a monetary policy shock as identified in the VAR might actually be a systematic response of the central bank to inflationary pressure. As the policy action only partially offsets inflation, it is followed by a modest increase in prices. Second, impulse response functions can only be computed for variables included in the VAR. But of course, researchers and policy makers are interested in the response of a much wider set of variables to monetary policy shocks. Examples of such variables that normally are not included in VARs are factor productivity, real wages, profits, or investment. Third, the variables contained in a VAR usually correspond directly to theoretical constructs. E.g. the construct "economic activity" may not be entirely

[1] I thank Thomas Maag for the preparation of this lecture note.

represented by GDP, industrial production, or any other directly observable measure. And again, the inclusion of additional variables is limited by the degrees-of-freedom problem.

Consequently, Bernanke et al. (2005, 389) ask: "Is it possible to condition VAR analyses of monetary policy on richer information sets, without giving up the statistical advantages of restricting the analysis to a small number of series?" Bernanke et al. (2005) present one approach which combines VAR and factor analysis. The resulting system is called a factor augmented vector autoregression, FAVAR.

2.2 Obtaining Factors: Principal Component Analysis, Factor Analysis

Recent evidence indicates that typically the information of a large set of macroeconomic variables X can be described adequately by a few factors F. Thus the solution to the degrees-of-freedom problem of VARs might simply be to add these few factors to the VAR. Following we first show how to distil common factors. In section 2.3 we then discuss how a standard VAR can be augmented by these common factors.

2.2.1 Derivation of principal components

Principal component analysis is an orthogonal transformation of the n-dimensional vector X into the n-dimensional set of uncorrelated principal components F:

$$\begin{aligned} F &= L'X \\ X &= LF \quad (L'L = I) \end{aligned} \tag{1}$$

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, F = \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}, L = \begin{pmatrix} l_{1,1} & \cdots & l_{1,n} \\ \vdots & \ddots & \vdots \\ l_{n,1} & \cdots & l_{n,n} \end{pmatrix}$$

I.e. each principal component is a linear combination of $x_1,...,x_n$:

$$f_i = l_{1,i}x_1 + l_{2,i}x_2 + ... + l_{n,i}x_n = l_i'X \tag{2}$$

The principal components f_i are constructed so that f_1 captures most of the variation in X, while f_2 captures most of the remaining variation, etc. Be S the variance-covariance matrix of X. We choose l_1 so that the first principal component $l_1'X$ has maximum variance:

Max. $VAR(f_1) = VAR(l_1'X) = l_1'Sl_1$ (3)
s.t. $l_1'l_1 = 1$

$$\begin{aligned} L &= l_1'Sl_1 - \lambda(l_1'l_1 - 1) \\ \frac{\delta L}{\delta l_1} &= 2Sl_1 - 2\lambda l_1 = 0 \\ &= (S - \lambda I)l_1 = 0 \end{aligned} \tag{4}$$

This is the eigenvalue-problem which has a non-trivial solution if $(S-\lambda I)$ is singular. Hence we have to choose the eigenvalue λ according to:

$$\det(S-\lambda I)=0 \tag{5}$$

The n-th order polynomial has n solutions for λ which are nonnegative, since S is a positive-semi definite variance-covariance matrix. We assume that the n eigenvalues are distinct and are ordered as follows:

$$\lambda_1 > \lambda_2 > ... > \lambda_n \geq 0$$

The variance of f_1 can be expressed in terms of eigenvalues:

$$VAR(f_1) = l_1'S'l_1 = l_1'\lambda I'l_1 = \lambda l_1'I'l = \lambda \tag{6}$$

since $l_1'l_1 = 1$. This expression is maximized if we choose $\lambda = \lambda_1$. l_1 is the eigenvector that corresponds to λ_1.

The maximization problem for the second principal component $l_2'X$ is complemented with a second constraint that ensures that f_1 and f_2 are uncorrelated:

$$\begin{aligned} \text{Max.} \quad & VAR(f_2) = VAR(l_2'X) = l_2'S'l_2 \\ \text{s.t.} \quad & l_2'l_2 = 1 \\ & Cov(f_2,f_1) = Cov(l_2'X, l_1'X) = E(l_2'XX'l_1) - E(l_2'X)E(l_1'X)' = l_2'Sl_1 \end{aligned} \tag{7}$$

Note that $Sl_1 = \lambda_1 l_1$, hence the constraint $l_2'Sl_1 = 0$ can be rewritten as $l_2'l_1 = 0$. The Lagrange-function takes the following form:

$$\begin{aligned} L &= l_1'Sl_1 - \lambda(l_2'l_2 - 1) - \upsilon(l_2'l_1) \\ \frac{\delta L}{\delta l_2} &= 2Sl_2 - 2\lambda l_2 - \upsilon l_1 = 0 \end{aligned} \tag{8}$$

We now show that $\upsilon = 0$. Premultiplication of the above expression with l_1' results in:

$$2l_1'Sl_2 - 2\lambda l_1'l_2 - \upsilon l_1'l_1 = 0 = 2l_1'Sl_2 - \upsilon \tag{9}$$

Since $l_1'l_1 = 1$, $l_1'l_2 = 0$. Thus:

$$2l_1'Sl_2 - \upsilon = 0 \tag{10}$$

Since $Cov(f_2,f_1)=0$, $Cov(f_1,f_2) = l_1'Sl_2 = 0$. Thus $\upsilon = 0$. The first order condition reduces to:

$$\begin{aligned} 2Sl_2 - 2\lambda l_2 &= 0 \\ (S-\lambda I)l_2 &= 0 \end{aligned} \tag{11}$$

To maximize the variance of f_2 we choose $\lambda = \lambda_2$. l_2 is the eigenvector that corresponds to λ_2. The remaining principal components $f_3 ... f_n$ can be derived in the same manner.

To summarize, $F = L'X$, where L is the matrix of column eigenvectors of S . The variance-covariance matrix Λ of F takes the following form:

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{pmatrix} \tag{12}$$

Note that:

$$\Lambda = L'SL$$
$$S = L\Lambda L', \text{ since } L'L = I$$
$$trace(\Lambda) = trace(L'SL) = trace(L'LS) = trace(S)$$

Where $trace(.)$ is the sum of diagonal elements of (.). Thus the sum of variance of principal components is equal to the sum of variance of the original variables in X . This again reflects that principal component analysis is a mathematical transformation without loss of information. Furthermore we can say that the i th principal component f_i explains the fraction $\lambda_i / (\sum_{j=1}^{n} \lambda_j)$ of total variation.

Usually, X is corrected by its mean, which does not alter eigenvalues and eigenvectors. The basic relationships of equations (1) become:

$$\begin{aligned} F &= L'(X - E(X)) \\ X &= LF + E(X) \end{aligned} \tag{13}$$

To conclude our brief exposition of principal component analysis, we define:

$$\begin{aligned} H &= L\Lambda^{0.5} \\ HH' &= S \end{aligned} \tag{14}$$

$$\begin{aligned} l_i^* &= \lambda_i^{0.5} l_i \\ l_i^{*}{}' l_i^* &= \lambda_i \end{aligned} \tag{15}$$

The principal components are scaled to have unit variance:

$$F^* = \Lambda^{-0.5} F \tag{16}$$

Thus:

$$X = HF^* \tag{17}$$

H is also called the matrix of component loadings, while F^* is called vector of principal factors.

2.2.2 Factor Analysis

The model of factor analysis is specified as follows (for simplicity we assume that $E(X) = 0$):

$$X = H^F F^F + e$$

$$H^F = \begin{pmatrix} h_{1,1}^F & \cdots & h_{1,p}^F \\ \vdots & \ddots & \vdots \\ h_{n,1}^F & \cdots & h_{n,p}^F \end{pmatrix}, F^F = \begin{pmatrix} f_1{}^F \\ \vdots \\ f_p{}^F \end{pmatrix}, e = \begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix} \tag{18}$$

where H^F is a (n x p) matrix of factor loadings, F^F a p-dimensional vector of factors, and e is a n-dimensional vector of error terms that describe the variance not explained by the $p < n$ factors. F^F is soften called matrix of common factors, while the error terms e are called specific factors. Usually, the following assumptions are imposed:

$$\begin{aligned} &E(f_i^F) = 0 \\ &Var(f_i^F) = E(f_i^{F2}) = 1 \\ &Cov(f_i^F, f_j^F) = E(f_i^F f_j^F) = 0, i \neq j \\ &E(e_i) = 0 \\ &E(e_i, e_j) = 0, i \neq j \\ &E(f_i^F, e_j) = 0 \end{aligned} \tag{19}$$

The variance-covariance matrix of X is given by:

$$S = E(H^F F^F + e)(H^F F^F + e)') = H^F H^F{}' + E \tag{20}$$

where E is the variance-covariance matrix of e. In particular:

$$Var(x_i) = h_{i,1}^2 + h_{i,2}^2 + ... + h_{i,p}^2 + Var(e_i) \tag{21}$$

The variance explained by common factors is called communality. Furthermore:

$$Cov(x_i, x_j) = h_{i,k} h_{j,k} \tag{22}$$

Note that if the variance-covariance matrix of e is required to be diagonal, the non-diagonal elements of S have to be explained fully by $H^F H^F{}'$. In addition it holds that:

$$Cov(x_i, f_j) = h_{i,j} \tag{23}$$

since $Var(x_i) = Var(f_i) = 1$ the above expression is equal to the correlation-coefficient between x_i and f_j.

The matrix of factor loadings can be created using different methods. The most obvious way is the so called principal component method. Here we simply select the first p principal factors from principal component analysis to be the p factors in the factor analysis model:

$$f_1 = f_1^*, ..., f_p = f_p^* \tag{24}$$

where f_i^* denotes the i-th element of the vector of principal factors F^*. The factor loadings are equal to the component loadings:

$$H^F = \begin{pmatrix} h_{1,1}^F & \cdots & h_{1,p}^F \\ \vdots & \ddots & \vdots \\ h_{n,1}^F & \cdots & h_{n,p}^F \end{pmatrix} = \begin{pmatrix} h_{1,1} & \cdots & h_{1,p} \\ \vdots & \ddots & \vdots \\ h_{n,1} & \cdots & h_{n,p} \end{pmatrix} = H_{(p)} \tag{25}$$

Note that $E = S - H_{(p)}H_{(p)}'$ is not diagonal.

2.3 FAVAR

A FAVAR(q) as specified by Bernanke et al. (2005) is given by:

$$\begin{bmatrix} F_t \\ Z_t \end{bmatrix} = B_0 + B_1 \begin{bmatrix} F_{t-1} \\ Z_{t-1} \end{bmatrix} + ... + B_q \begin{bmatrix} F_{t-q} \\ Z_{t-q} \end{bmatrix} + Su_t \tag{26}$$

$$F_t = \begin{pmatrix} f_{1,t} \\ \vdots \\ f_{p,t} \end{pmatrix}, Z_t = \begin{pmatrix} z_{1,t} \\ \vdots \\ z_{k,t} \end{pmatrix}, B_0 = \begin{pmatrix} c_1^{(0)} \\ \vdots \\ c_{p+k}^{(0)} \end{pmatrix}, B_i = \begin{pmatrix} a_{1,1}^{(i)} & \cdots & a_{1,k}^{(i)} \\ \vdots & \ddots & \vdots \\ a_{p+k,1}^{(i)} & \cdots & a_{p+k,k+k}^{(i)} \end{pmatrix} \text{ for } i = 1...q \tag{27}$$

$$u_t = \begin{pmatrix} u_{1,t} \\ \vdots \\ u_{p+k,t} \end{pmatrix}, Eu_t = 0, Eu_t u_t' = V, Eu_t u_s' = 0 \text{ if } t \neq s \tag{28}$$

where Z is a vector of k observable economic variables that are assumed to drive the system. F is the vector of p unobservable factors that summarize additional relevant information X. In principle, the above system is an ordinary VAR(q) in Z and F. The estimation results, particularly the explanatory power of the additional factors can be directly compared to a standard VAR that only covers Z. Of course, if the true data generating process follows an FAVAR then the coefficient estimates of a standard VAR will be biased.

The above equation can not be directly estimated since F_t is unobservable. Bernanke et al. (2005) assume that the relationship between informational time series X, common factors F, and observed variables Z is given by the following dynamic factor model:

$$X_t = H^F F_t + H^Z Z_t + e_t \tag{29}$$

$$H^F = \begin{bmatrix} h_{1,1}^F & \cdots & h_{1,p}^F \\ \vdots & \ddots & \vdots \\ h_{n,1}^F & \cdots & h_{n,p}^F \end{bmatrix}, H^Z = \begin{bmatrix} h_{1,1}^Z & \cdots & h_{1,k}^Z \\ \vdots & \ddots & \vdots \\ h_{n,1}^Z & \cdots & h_{n,k}^Z \end{bmatrix} \quad (30)$$

where X is an n-dimensional vector, and $n >> p + k$. H^F is a matrix of factor loadings. Equation (29) states that both F and Z represent common factors that drive X.

Bernanke et al. (2005) consider two approaches to estimate the FAVAR, a two-step principal components approach and a one step approach which estimates (26) and (29) jointly. Following we focus on the two-step procedure.

In the first step, the first $p + k$ principal components of X are calculated. $\hat{F}_t$ is then recovered as part of the space covered by (F_t', Z_t') that is not covered by Z_t alone. Bernanke et al. (2005) note that if n is large and if the number of principal components used is at least as large as the true number of factors, the principal components consistently recover the space spanned by (F_t', Z_t'). To identify equation (29), Bernanke et al. (2005) impose the following restrictions:

$$F = (F_1, F_2, ..., F_T)' = \begin{pmatrix} f_{1,t=1} & f_{2,t=1} & \cdots & f_{p,t=1} \\ f_{1,t=2} & f_{2,t=2} & & f_{p,t=2} \\ & & & \\ f_{1,t=T} & f_{2,t=T} & \cdots & f_{p,t=T} \end{pmatrix}$$

$$F'F = \begin{pmatrix} f_{1,t=1} & f_{1,t=2} & \cdots & f_{1,t=T} \\ f_{2,t=1} & f_{2,t=2} & & f_{2,t=T} \\ \vdots & & & \vdots \\ f_{p,t=1} & f_{p,t=2} & \cdots & f_{p,t=T} \end{pmatrix} \begin{pmatrix} f_{1,t=1} & f_{2,t=1} & \cdots & f_{p,t=1} \\ f_{1,t=2} & f_{2,t=2} & & f_{p,t=2} \\ & & & \\ f_{1,t=T} & f_{2,t=T} & \cdots & f_{p,t=T} \end{pmatrix} = \begin{pmatrix} 1*T & 0 & \cdots & 0 \\ 0 & 1*T & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1*T \end{pmatrix} \quad (31)$$

Typically, F and Z will be correlated, i.e. any of the linear combinations resulting in F_t potentially involves Z. Hence it is not possible to simply estimate (26) and to identify the monetary policy shock. It is necessary to remove the direct dependency of F_t on Z_t, which will be discussed in the next section.

In the second step, equation (26) is estimated by OLS. Note that since part of the regressors have been estimated conventional test statistics are rendered useless.

2.4 Empirical Results of Bernanke et al. (2005)

Bernanke et al. (2005) consider three different model specifications. In the benchmark model only the Federal Funds rate (FFR) is treated as observable, i.e. Z_t is a scalar. The authors argue that in practice even variables such as output or price level cannot be directly observed, since they are subject to revisions. Furthermore Bernanke et al. (2005) emphasize that theoretical concepts are seldom reflected in specific time series. E.g. the variable "output" that frequently appears in monetary policy feedback rules does rather correspond to a latent and broad measure of economic activity than to one single time series (namely GDP). Consequently Bernanke et al. (2005) include 120 variables in X_t. The balanced panel consists of monthly time series ranging from January

1959 to August 2001. The sectors covered are real output and income, employment and hours, consumption, housing starts and sales, real inventories and orders, stock prices, exchange rates, interest rates, money and credit aggregates, price indices, hourly earnings. The three most important factors are added to the benchmark FAVAR.

Bernanke et al. (2005) remove the linear relation between F_t and Z_t by estimating the following regression:

$$F_t = a\widehat{F}_t + bFFR_t + e_t \qquad (32)$$

where $\widehat{F}_t$ is an estimate of all common components excluding the Federal Funds rate. Bernanke et al. (2005) calculate $\widehat{F}_t$ from a subset of X_t that contains variables that are unlikely to be contemporaneously affected by the Fed Funds rate. The vector of common factors not dependent on Z_t is given by $F_t^{corr} = F_t - bFFR_t$.

Bernanke et al. (2005) consider two alternative specifications: A standard VAR that includes three endogenous variables industrial production (IP), CPI, and the Federal Funds rate, and an alternative FAVAR that expands the standard three-variable VAR with one unobservable factor. All models include 13 lags. Bernanke et al. (2005, 406) note that the estimation results are robust against changes in the number of lags and factors included. In each case the monetary policy shock is identified applying a Choleski decomposition. This recursive ordering imposes the restriction that the factors F_t are not contemporaneously affected by monetary policy shocks.

The response of FFR, IP, and CPI to a contractionary 25 basis points monetary policy shock are shown in Figure 1. All responses are measured in standard deviations. At first sight the impulse-response functions of the two FAVAR specifications are quite similar, whereas the VAR specification yields distinct results. According to the VAR, a monetary policy shock has persistent effects on industrial production which is inconsistent with the notion of long run neutrality of money. In addition, the price level rises rather than declines. These shortcomings do not appear in the results of the FAVAR anymore. Here the advantage of the FAVAR approach becomes visible: It offers a systematic way to select relevant additional information to be included in the model. E.g. we could also include commodity prices to correct for the wrong direction of the price response, but then again it would be difficult to justify the selection of this variable.

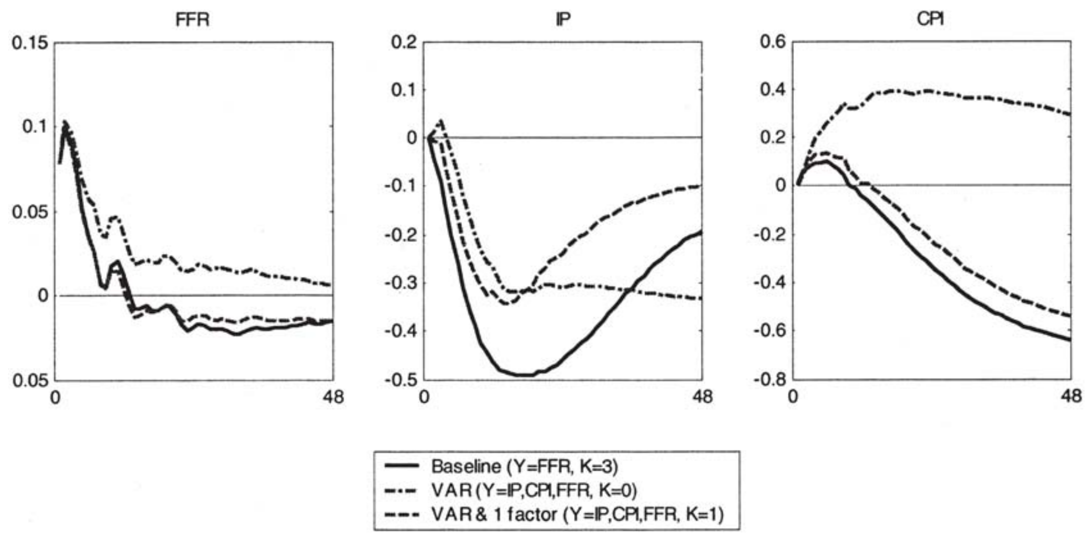

Figure 1: Impulse-Response Functions to a Monetary Policy Shock. Source: Bernanke (2005, 406).

Given the relationship between F_t and X_t as specified in (18) it is also possible to calculate impulse response functions for each variable in X_t. Figure 2 displays impulse-response function and the corresponding 90% significance interval for 20 elements of X_t. The impulse-response functions were generated from the benchmark FAVAR.

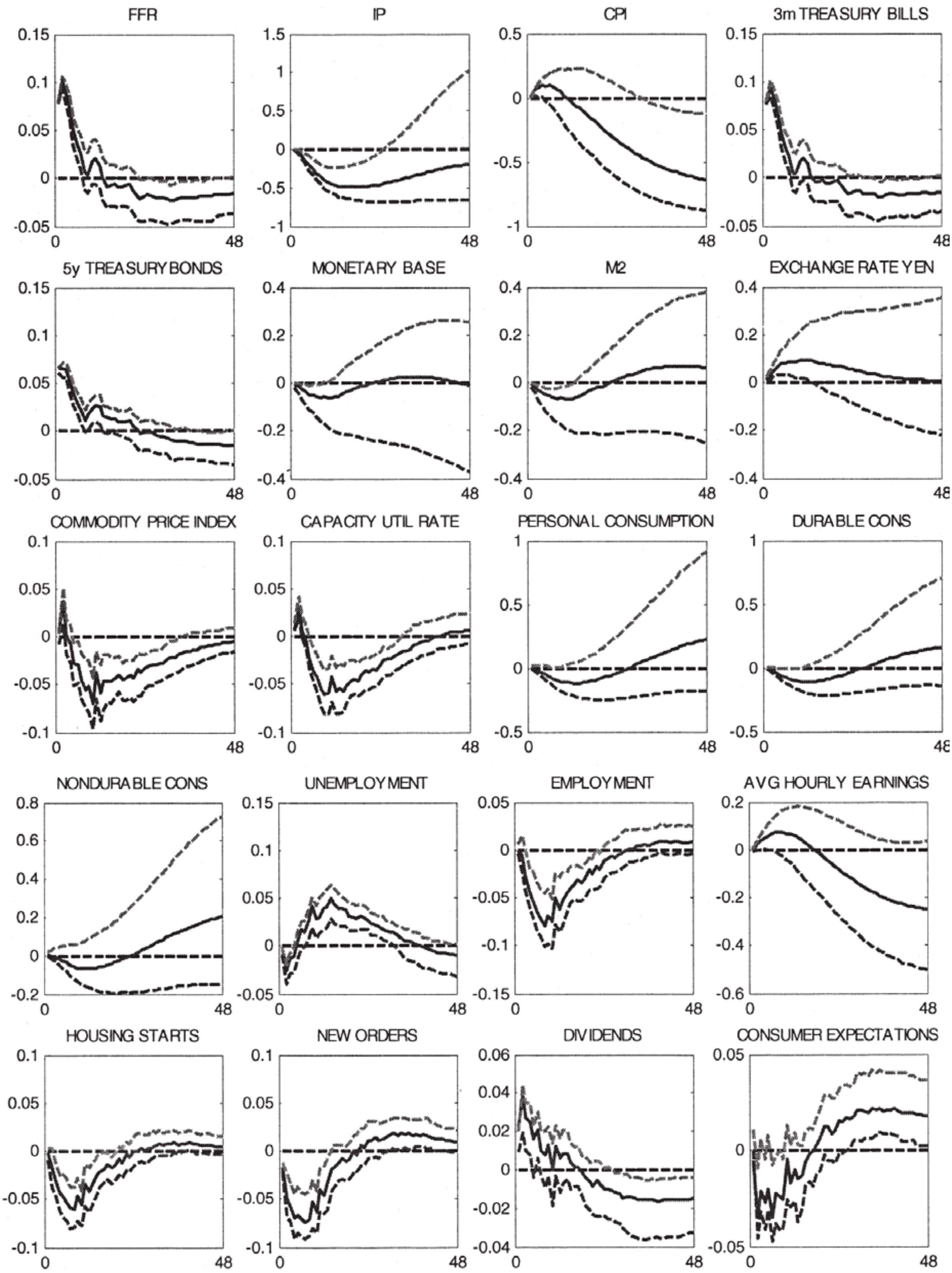

Figure 2: Impulse-Response Functions of Elements of X. Source: Bernanke et al. (2005, 408).

3 The Lucas Programme

3.1 The Lucas Critique: An Illustration

Source: Rudebusch (2005), Lucas (1976)

In a seminal paper, Lucas (1976) stresses that coefficients of reduced form models depend on expectations of agents and are thus unlikely to remain stable if policymakers change their behaviour. Lucas (1976, 40) considers the following reduced form model of an economy:

$$Y_{t+1} = F(Y_t, X_t, \Theta(\Lambda), u_t) \tag{33}$$

where Y is a vector of economic variables, X is a vector of policy instruments, $\Theta(.)$ is a vector of behavioural parameters and u is a vector of reduced form shocks. A policy rule is given by:

$$X_t = G(Y_t, \Lambda, \varepsilon_t) \tag{34}$$

where Λ is a vector of policy rule parameters, and ε is a vector of random shocks. Lucas (1976, 40) states that "A change in policy (in Λ) affects the behaviour of the system in two ways: first by altering the time series behaviour of X_t; second by leading to modification of the behavioural parameters $\Theta(\Lambda)$ governing the rest of the system." This second effect is related to changed expectations of agents that respond to the change in a policy rule. Following we illustrate this expectational effect. Consider a slightly modified version of the New Keynesian model that has been discussed in the previous lecture. The model is a simplified version of the lagged expectations model presented in Rudebusch (2005):

$$\begin{aligned} y_t &= \alpha y_{t-1} - \vartheta(E_{t-1}i_t - \pi_{t-1}) + \varepsilon_{1,t} \\ \pi_t &= \pi_{t-1} + \gamma y_t + \varepsilon_{2,t} \\ i_t &= \lambda y_t + \varphi \pi_t + \varepsilon_{3,t} \end{aligned} \tag{35}$$

Again, the first equation is an IS curve and the second equation can be interpreted as a Phillips-curve relationship. The third equation defines a Taylor-rule like monetary policy reaction function, where ε_t is a monetary policy shock. We can rewrite the above system in expectations:

$$\begin{aligned} E_{t-1}y_t &= \alpha y_{t-1} - \vartheta(E_{t-1}i_t - \pi_{t-1}) \\ E_{t-1}\pi_t &= \pi_{t-1} + \gamma E_{t-1}y_t \\ E_{t-1}i_t &= \lambda E_{t-1}y_t + \varphi E_{t-1}\pi_t \end{aligned} \tag{36}$$

Solving for the expected interest rate results in:

$$\begin{aligned} E_{t-1}i_t &= \lambda\alpha y_{t-1} - \lambda\vartheta(E_{t-1}i_t - \pi_{t-1}) + \varphi\pi_{t-1} + \varphi\gamma\alpha y_{t-1} - \varphi\gamma\vartheta(E_{t-1}i_t - \pi_{t-1}) \\ &= (\lambda\alpha + \gamma\varphi\alpha)y_{t-1} + (\lambda\vartheta + \varphi + \gamma\varphi\vartheta)\pi_{t-1} - (\lambda\vartheta + \gamma\varphi\vartheta)E_{t-1}i_t \\ E_{t-1}i_t &= \frac{\lambda\alpha + \gamma\varphi\alpha}{1 + \lambda\vartheta + \gamma\varphi\vartheta} y_{t-1} + \frac{\lambda\vartheta + \varphi + \gamma\varphi\vartheta}{1 + \lambda\vartheta + \gamma\varphi\vartheta}\pi_{t-1} \end{aligned} \tag{37}$$

Hence we get the following reduced form solution for output y_t:

$$\begin{aligned} y_t &= \alpha y_{t-1} - \vartheta\left(\frac{\lambda\alpha + \gamma\varphi\alpha}{1 + \lambda\vartheta + \gamma\varphi\vartheta} y_{t-1} + \frac{\lambda\vartheta + \varphi + \gamma\varphi\vartheta}{1 + \lambda\vartheta + \gamma\varphi\vartheta}\pi_{t-1} - \pi_{t-1}\right) + \varepsilon_{1,t} \\ &= \frac{\alpha(1 + \lambda\vartheta + \gamma\varphi\vartheta) - \vartheta(\lambda\alpha + \gamma\varphi\alpha)}{1 + \lambda\vartheta + \gamma\varphi\vartheta} y_{t-1} - \vartheta\frac{\lambda\vartheta + \varphi + \gamma\varphi\vartheta - (1 + \lambda\vartheta + \gamma\varphi\vartheta)}{1 + \lambda\vartheta + \gamma\varphi\vartheta}\pi_{t-1} + \varepsilon_{1,t} \\ &= \frac{\alpha}{1 + \lambda\vartheta + \gamma\varphi\vartheta} y_{t-1} - \frac{\vartheta(\varphi - 1)}{1 + \lambda\vartheta + \gamma\varphi\vartheta}\pi_{t-1} + \varepsilon_{1,t} \end{aligned} \tag{38}$$

The reduced form representation of output y_t depends on the policy parameters λ and φ. Consequently the reduced form parameter estimates of this equation will not be stable over different policy regimes.

Rudebusch (2005) notes that for the US several recent papers reject the stability of the estimated monetary policy rule. This suggests that the Lucas critique is potentially relevant in the field of monetary policy analysis. Interestingly however the widely used VARs that do not incorporate expectations often are stable empirically. Rudebusch (2005) tries to reconcile these contradictory findings by examining the importance of the quantitative extent of parameter shifts in the monetary policy rule. The results of Rudebusch (2005) indicate that while the theoretical relevance of the Lucas critique is uncontested, the empirical relevance in this specific context seems to be minor.

3.2 The Lucas Programme: A New Role of Empirical Analysis

Source: Christiano et al. (1998), Favero (2001, 3.4), Rudebusch (2005)

The Lucas-critique suggests using structural models that incorporate "deep parameters" (such as taste, technology) and expectations to identify optimal policy reaction. Consequently the role of empirical analysis changes: Empirical analysis should now provide evidence on the stylized responses of macroeconomic variables to policy shocks. On the basis of these stylized facts, competing structural models can be assessed. Lucas (1980) proclaims that economists need to "... test them (models) as useful imitations of reality by subjecting them to shocks for which we are fairly certain how actual economies or parts of economies would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder questions."

Christiano et al. (1998) operationalize this idea using monetary policy shocks, which they regard to be particularly useful due to distinct reactions of different models to these shocks. Christiano et al. (1998, 3) suggest a three-step procedure:

> "- First, one isolates monetary policy shocks in actual economies and characterizes the nature of the corresponding monetary experiments.
> - Second, one characterizes the actual economy's response to these monetary experiments.
> - Third, one performs the same experiments in the model economies to be evaluated and compares the outcomes with actual economies' responses to the corresponding experiments."

Christiano et al. (1998) identify monetary policy shocks using the VAR-approach. Stylized facts are derived from the shape of the estimated impulse-response functions to monetary policy shocks. The methodology and some empirical results have been presented in lecture 1.

4 Leeper / Zah: The Concept of Modest Policy Changes

Source: Leeper and Zah (2003), Jordan et al. (2002)

4.1 The concept of modest policy interventions

Motivated by the Lucas critique, Leeper and Zah (2003) introduce the concept of modest policy intervention. The authors define a modest policy intervention as an intervention that does not significantly shift agents' beliefs about the policy regime. Thus modest policy interventions can be captured with reduced form linear VARs, since they do not cause structural parameters to change. Leeper and Zah (2003) simulate a theoretical model to assess what a modest intervention actually is.

In short, the central results of Leeper and Zah (2003, 1675) are:

- "modest policy interventions may have substantial impacts without generating important expectations-formation effects
- a small but persistent intervention is more likely to destabilize a linear model than is a large but fleeting intervention
- An inter-regime policy is modest, when the proposed statistic is close to its mean. I.e. the modesty statistic displays the direct effect relative to their historical variation. If direct effects are unusually large, which may or may not trigger changes in beliefs, policy shocks are labelled immodest."

4.2 Direct and indirect effects of monetary policy

Leeper and Zah (2003, 1678) use a model that allows monetary policy to switch randomly between two regimes. The price setting equation follows the idea of costly price adjustment, where the adjustment costs are captured by α:

$$p_t = \alpha p_{t-1} + (1-\alpha)(1-\alpha\beta)E_{t-1}\sum_{j=0}^{\infty}(\alpha\beta)^j m_{t+j}$$

$$y_t = \left(m_t - \frac{1-\alpha}{1-\alpha L}E_{t-1}\frac{1-\alpha\beta}{1-\alpha\beta L^{-1}}m_t\right)$$

$$m_t = g_t + m_{t-1}$$

In this framework, expected monetary policy affects both prices and output. Note that if $\alpha = 0$ then output is only affected by unexpected monetary policy, since $y_t = m_t - E_{t-1}m_t$. Monetary policy switches between two regimes R_1, R_2 with determined transition probabilities. A regime change will shift the mean, the dynamics and the variance of money growth g_t:

$$g_t = \mu(R_t) + \nu(R_t)g_{t-1} + \sigma(R_t)\varepsilon_t^s$$

where a ε_t^s denotes a monetary policy shock with unit variance. The agents are modelled to behave as rational Bayesian updaters. While it is assumed that agents observe the history of money growth, they do not directly observe the realizations of the regime. Hence they use the observed information set $\Omega_{t-1} = \{p(R_0), m_0, g_0, g_1, ..., g_{t-1}\}$, where $p(R_0)$ is the agents belief about the regime at time $t = 0$, to infer the probability of policy being in each of the two regimes, $p(R_{t-1} = R^s | \Omega_{t-1}), s = 1,2$.

Given a fixed regime, the theoretical model has a linear reduced form representation which can be captured in a SVAR:

$$A_0 Z_t = C_0 + A_1 Z_{t-1} + A_2 Z_{t-2} + ... + A_q Z_{t-q} + \varepsilon_t$$

If A_0 is non-singular the above system can be rewritten as an MA-model in structural shocks:

$$Z_t = \mu + \varepsilon_t + \Psi_1 \varepsilon_{t-1} + \Psi_2 \varepsilon_{t-2} + \Psi_3 \varepsilon_{t-3} + \dots \quad (39)$$

The K-period forecast of Z_{T+K} conditional on a given path of monetary policy interventions $I_T = \{\tilde{\varepsilon}^s_{T+1},...,\tilde{\varepsilon}^s_{T+K}\}$ under regime 1 given all available information at time T is:

$$\begin{aligned} Z_{T+K} &= E(Z_{T+K} \mid I_t, \Omega_T, R_t = R^1, t = T+1...K) + \tilde{\varepsilon}_{T+K} + \Psi_1 \tilde{\varepsilon}_{T+K-1} + ... + \Psi_{K-1} \tilde{\varepsilon}_{T+1} \\ \tilde{\varepsilon}_t &= \begin{pmatrix} 0 \\ \vdots \\ \varepsilon^s_t \\ \vdots \\ 0 \end{pmatrix}, t = \{T+1,...,T+K\} \end{aligned} \quad (40)$$

In this setting, Leeper and Zah (2003) introduce the notion of direct effects ${\eta^s}_{T+K}$ of I_t. Direct effects are measured relative to a forecast with no policy interventions, i.e. with $I_t = \{0\}$:

$$\begin{aligned} {\eta^s}_{T+K} &= \tilde{\varepsilon}_{T+K} + \Psi_1 \tilde{\varepsilon}_{T+K-1} + ... + \Psi_{K-1} \tilde{\varepsilon}_{T+1} \\ &= E(Z_{T+K} \mid I_t, \Omega_T, R_t = R^1, t = T+1...K) - E(Z_{T+K} \mid \Omega_T, R_t = R^1, t = T+1...K) \end{aligned} \quad (41)$$

Given a variable regime, the economy is nonlinear and can no longer be correctly captured by the linear system in (39). An intervention may now cause changes in agents' beliefs about the policy regime, which again affects agent's expectations of future policy and their optimal choices. Total effects of the policy intervention I_T on Z_{T+K} thus entail direct effects and expectations formation effects. Total effects are equal to:

$${\eta^{Total}}_{T+K} = E(Z_{T+K} \mid I_t, \Omega_T) - E(Z_{T+K} \mid \Omega_T, R_t = R^1, t = T+1...K) \quad (42)$$

Of course, if expectations formation effects are minor relative to direct effects, then the linear model will deliver a relatively accurate description of the economy.

4.3 Capturing the modesty of policy shocks

Leeper and Zah (2003, 1680) suggest the following statistic to measure the modesty of policy shocks:

$$\eta^{*s}{}_{T+K} = \begin{pmatrix} \sum_{s=0}^{K-1} \psi_{1,j}(s)^2 & 0 & \cdots & 0 \\ 0 & \sum_{s=0}^{K-1} \psi_{2,j}(s)^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{s=0}^{K-1} \psi_{k,j}(s)^2 \end{pmatrix}^{-1} \eta^{s}{}_{T+k} \quad (43)$$

where $\psi_{i,j}(s)$ as the row i, column j element of Ψ_s. Here the error term ε_t^s is the jth element of ε_t. Hence the $\psi_{i,j}(s)$ quantifies the consequence of a one unit increase in the error term ε_t^s for the value of $z_{i,t+s}$, all other innovations held constant. $\eta^{*s}{}_{T+K}$ thus corresponds to $\eta^{s}{}_{T+K}$ with all elements being scaled by the standard deviation of the impact multipliers (i.e. scaled by the standard deviation of the respective direct effect assuming that the structural errors have unit variance). $\eta^{*s}{}_{T+K}$ follows a normal distribution with unit variance.

Leeper and Zah (2003, 1680) define: "An intervention is modest if over a specified forecast horizon k and for the ith variable in Z,

$$|e\eta^{*s}{}_{T+K}| \quad (44)$$

is not close to 2 (standard deviations), where e is a conformable row vector of zeros with unity in the ith column." The modesty statistic thus reports how unusual the impact of monetary policy intervention $I_T = \{\tilde{\varepsilon}^s_{T+1},...,\tilde{\varepsilon}^s_{T+K}\}$ is relative to the typical size of direct effects. Intuitively we see that if direct effects are conformable with typical (historical) effects of monetary policy, there is no reason for agents to change their beliefs about the prevailing regime, and expectations-formation effects are likely to be insignificant.

Leeper and Zah (2003) apply the theoretical considerations in an empirical setting. While the true model of monetary policy $S_t = g(\Omega_t)$ is unobservable, the econometrically captured model of monetary policy is $S_t = f(\mathrm{O}_t) + \varepsilon_t^s$, where $f(.)$ is a linear function and $\mathrm{O}_t \subset \Omega_t$ is the state of the economy as observed by private agents. A policy regime is a specific choice of $g(.)$, as such it is reflected in the functional form $f(.)$ of and the stochastic process of ε_t^s. Monetary policy is captured in a VAR, where $Z_t = (\text{real output, cpi, unemployment, short term rate, M2, commodity prices})'$. The short term rate is considered to be the monetary policy instrument. The impulse response functions which capture direct effects of a monetary policy shock are identified by a Choleski decomposition. Leeper and Zah (2003, 1698) find that "accurately identified linear econometric models can reliably predict the impacts of modest policy interventions".

4.4 Modest policy shocks in the SNB SVAR model

To assess the modesty of policy shocks, two measures are employed:

$$\eta^{12} = \frac{1}{12}\sum_{i=1}^{12} \varepsilon^s_{T+i} \quad (45)$$

$$Q^{12} = \sum_{i=1}^{12} (\varepsilon^s_{T+i})^2 \quad (46)$$

The mean of simulated monetary policy shocks indicates whether the policy shock is systematically biased. The sum of the squared policy shocks follows the idea of Leeper and Zah (2003) and has to be assessed relative to its historical mean. Under the assumption that $\varepsilon_t^s \sim iidn(0,1)$, $\eta^{12} \sim N(0,\sqrt{1/T})$, and $Q^{12} \sim \chi^2(12 \text{ df.})$.

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