I) Chapter: Complement to lecture 3

II) Title: Structural Time Series Models and Output Gap Estimation

III) Contents: Structural time series models, output gap estimation

IV) Documentation:

- Clark, Peter K. (1987). The Cyclical Component of U.S. Economic Activity, *The Quarterly Journal of Economics*, Vol. 102, p. 797
- Gerlach, Stefan and Frank Smets (1999). Output gaps and monetary policy in the EMU area, *European Economic Review*, Vol. 43, p. 801-812.
- Harvey, Andrew C. (1985). Trends and Cycles in Macroeconomic Time Series, *Journal of Business and Economic Statistics*, Vol. 3 p. 216
- Harvey, Andrew C. (1993). *Time Series Models*, MIT Press
- Harvey, Andrew C. and A. Jaeger (1993) Detrending, stylized facts and the business cycle, *Journal of Applied Econometrics*, Vol. 8, p. 231-247.
- Kuttner, Kenneth N. (1994). Estimating Potential Output as a Latent Variable, *Journal of Business & Economic Statistics*, Vol. 12, No. 3.
- Orphanides, Athanasios and Simon van Norden (1999). The Reliability of Output Gap Estimates in Real Time, *working paper*.
- Orphanides, Athanasios and Simon van Norden (2002). The Unreliability of Output-Gap Estimates in Real Time, *The Review of Economics and Statistics*, Vol. LXXXIV, No. 4.
- Watson, Mark M. (1986). Univariate Detrending Methods with Stochastic Trends, *Journal of Monetary Economics*, Vol. 18, p. 49

1.) Introduction

A straightforward application of the state-space framework is *structural time series modelling (STSM)*. Harvey uses this expression to explicitly state that the coefficients and variables in the models he suggests for detrending, seasonal adjustment and extraction of cyclical components have a well defined economic meaning. He contrasts this/his models to the standard ARIMA framework where it is hard to give an economic interpretation of what one is exactly doing.
We will first provide a quick overview of the STS models and in a later step show how those can be used and have been used to extract estimates of the output gap.

2.) Structural Time Series Models

We can use state-space modelling to decompose a time series into trend, cyclical component and seasonal effects.

2.1) Trend

Let us first consider modelling a trend component of the time series y_t. The most general state-space model is the *local linear trend* defined by the observation equation

$$y_t = \mu_t + \varepsilon_t \tag{0.1}$$

where $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$ and the trend μ_t has the state equation

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t \\ \beta_t &= \beta_{t-1} + \zeta_t \end{aligned} \tag{0.2}$$

where $\eta_t \sim N\left(0, \sigma_\eta^2\right)$ and $\zeta_t \sim N\left(0, \sigma_\zeta^2\right)$. Through the state β_t the slope of the trend is allowed to change over time.

This model includes other concepts of a trend as special cases:

- random walk with drift if $\sigma_\zeta^2 = 0$,
- random walk if we drop the state β_t,
- deterministic linear trend $\mu_t = \alpha + \beta t$ if $\sigma_\varepsilon^2 = \sigma_\zeta^2 = 0$.
- Hodrick-Prescott filter if $\sigma_\eta^2 = 0$ and if we set the signal to noise ratio equal to

$q_\zeta = \dfrac{\sigma_\zeta^2}{\sigma_\varepsilon^2} = \dfrac{1}{\lambda}$, where λ is the usual HP smoothing parameter: e.g. for quarterly data $\lambda = 1600$ or equivalently $q_\zeta = 0.000625$.

2.2) Seasonality

We can extend the model (0.1) & (0.2) by adding the state process γ_t for the seasonal pattern

$$y_t = \mu_t + \gamma_t + \varepsilon_t \tag{0.3}$$

Suppose y_t is a quarterly time series, then we could specify γ_t either as

$$\gamma_t = -\gamma_{t-1} - \gamma_{t-2} - \gamma_{t-3} \tag{0.4}$$

or as

$$\gamma_t = -\gamma_{t-1} - \gamma_{t-2} - \gamma_{t-3} + \omega_t \tag{0.5}$$

The former formulation imposes that the seasonal components add to zero over one year. The latter formulation allows the seasonal pattern to change over time, since it sets the sum of the seasonal components of one year equal to the white noise process $\omega_t \sim N\left(0, \sigma_\omega^2\right)$.

2.3) Cycles

Extending the model (0.3) & (0.2) & (0.5) by adding a cyclical component ψ_t, yields observation equation

$$y_t = \mu_t + \psi_t + \gamma_t + \varepsilon_t \tag{0.6}$$

There are various ways for modelling the cyclical component. One could consider using an AR(2) process

$$\psi_t = \rho_1 \psi_{t-1} + \rho_2 \psi_{t-2} + \kappa_t \tag{0.7}$$

where $\kappa_t \sim N\left(0, \sigma_\kappa^2\right)$. Alternatively one could go to the frequency domain and model the cycle as a stochastic cycle

$$\begin{aligned} \psi_t &= \rho \cos \lambda_c \psi_{t-1} + \rho \sin \lambda_c \psi_{t-1}^* + \kappa_t \\ \psi_t^* &= -\rho \sin \lambda_c \psi_{t-1} + \rho \cos \lambda_c \psi_{t-1}^* + \kappa_t^* \end{aligned} \tag{0.8}$$

where ρ is a damping parameter, λ_c denotes the frequency and ψ_t^* is only introduced to construct ψ_t. This model includes the deterministic cycle

$$\psi_t = \alpha \cos \lambda_c t + \beta \sin \lambda_c t$$

as a special case.

3.) Output gap estimates

An obvious time series for the application of the structural time series models is the real GDP. In what follows we will present the paper by Orphanides and van Norden (2002) on "The Unreliability of Output-Gap Estimates in Real Time."
Orphanides and van Norden (OvN hereinafter) analyse and compare different methods for calculating the output gap. The innovation of their paper lies in the fact that they use real time data to evaluate the performance of various methods for calculating the output gap. OvN observe that "three distinct issues complicate measurement of the output gap in real time. First, output data may be revised [...]. Second, as data on output in subsequent quarters become available, hindsight may clarify our position in the business cycle even in the absence of the data revision. Third, the arrival of new data may instead make us revise our model of the economy, which in turn revises our estimated output gaps."

3.1) Alternative detrending methods

The log of real (and seasonally adjusted) GDP q_t is decomposed

$$q_t = \mu_t + z_t \tag{0.9}$$

into trend μ_t and cycle z_t.
The simplest method is to assume a *linear trend*

$$q_t = \alpha + \beta t + z_t \tag{0.10}$$

where the error term is assumed to be the cycle. Slightly more sophisticated methods are the *quadratic trend* and the *breaking trend*. OvN assume that the moment of the break is known and that it has happened in 1973.
The next class of detrending methods is based on the structural time series models discussed in section two. Probably the most commonly used method to obtain an output gap is the (univariate) *Hodrick-Prescott Filter*. OvN apply the HP filter with a smoothing parameter of $\lambda = 1600$ in order to obtain an estimate for the trend of GDP. More complex univariate models studied in OvN are those by Watson (1986), Harvey (1985) and Clark (1987). In what OvN call the Watson model the trend is specified as a random walk with drift and the cycle as an AR(2) process

$$\begin{aligned} \mu_t &= \delta + \mu_{t-1} + \eta_t \\ z_t &= \rho_1 z_{t-1} + \rho_2 z_{t-2} + \varepsilon_t \end{aligned} \tag{0.11}$$

and the Harvey-Clark model is simply a local linear trend model with an AR(2) cycle

$$\begin{aligned} \mu_t &= g_t + \mu_{t-1} + \eta_t \\ g_t &= g_{t-1} + \upsilon_t \\ z_t &= \rho_1 z_{t-1} + \rho_2 z_{t-2} + \varepsilon_t \end{aligned} \tag{0.12}$$

where η_t, υ_t and ε_t are uncorrelated white noise disturbances.
In an attempt to improve output gap estimates it has often been suggested to include information from other variables that are linked to the output gap in the estimation of the it. OvN cosider the model by Kuttner (1994) where a Phillips curve is added to the Watson model

$$\Delta\pi_t = \xi_1 + \xi_2 \Delta q_t + \xi_3 z_{t-1} + e_t + \xi_4 e_{t-1} + \xi_5 e_{t-2} + \xi_6 e_{t-3} \tag{0.13}$$

and the Gerlach and Smets (1999) model where a Phillips curve of the following form is added to the Harvey-Clark model

$$\Delta\pi_t = \phi_1 + \phi_2 z_t + e_t + \phi_3 e_{t-1} + \phi_4 e_{t-2} + \phi_5 e_{t-3} \qquad (0.14)$$

where e_t is a white noise error term which is assumed to be uncorrelated with any other error terms in the Gerlach Smets model but is assumed to be correlated with the trend error term in the Kuttner model.

3.2) Data sources and revision concepts

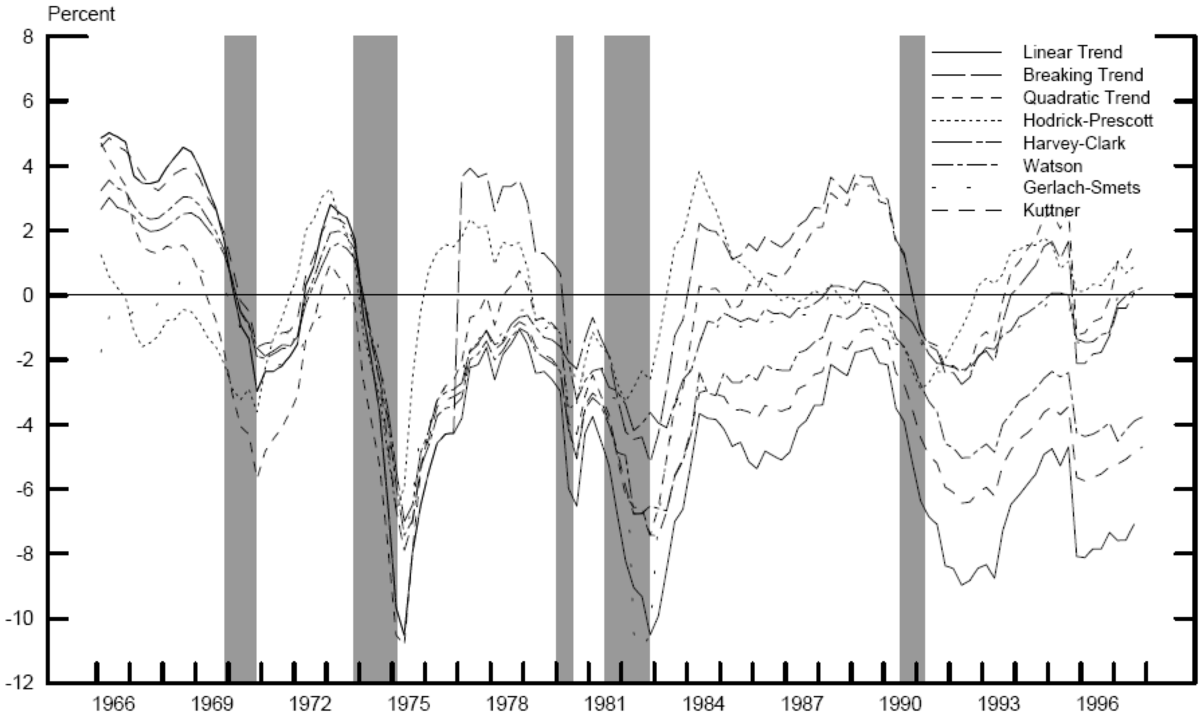
OvN use quarterly data from a real-time dataset compiled by Croushore and Stark[1]. OvN publish estimated output gaps for 1966Q1-1997Q4. In order to measure the impact of data revision, OvN use a final estimate of the output gap as a benchmark.
The *final estimate* of the output gap is made by using the (at their time) last available vintage of data from 2000Q1. (Note: In the Croushore and Stark dataset, this corresponds to vintage 2000Q2, which contains data up to 2000Q1.)
"The *real-time estimate* of the output gap is constructed in two stages. First, we detrend each and every vintage of data available to construct an ensemble of output gap series. That is in every quarter we apply the detrending method with data as available during that quarter. Next, we use these different vintages to construct a new series, which consists of the latest available estimate of the output gap for each point in time."
The difference between the real-time and the final estimate may result from different sources. OvN compute a quasi-real (-time) estimate of the output gap in order to isolate the impact of data revision. The *quasi-real estimate* is a rolling estimate based on the final vintage data series. "That is, the gap at period t is calculated using only observations 1 through t to estimate the long-run trend and the deviations around it."
Further, for the unobserved components models OvN use smoothed estimates (based on the final vintage) of the state process as final estimates and filtered estimates of the state process as *quasi-final estimates*. "The difference between the quasi-final and final series reflects the importance of ex post information in estimating the output gap given the parameter values of the process generating output."
OvN use the standard errors provided by the Kalman filter and smoother in order to construct 95% confidence intervals for the output gap estimates. These standard errors capture the effect of final-quasi final revision but ignore the effect of parameter uncertainty. Therefore OvN also compute Ansley-Kohn errors, which represent an approximate measure taking into account the parameter uncertainty as well.

[1] This dataset covers more than just the GDP and CPI and is available online on http://www.phil.frb.org/econ/forecast/reaindex.html.

3.3) Results

Figure 1

Real-Time Estimates of the Business Cycle



Final Estimates of the Business Cycle

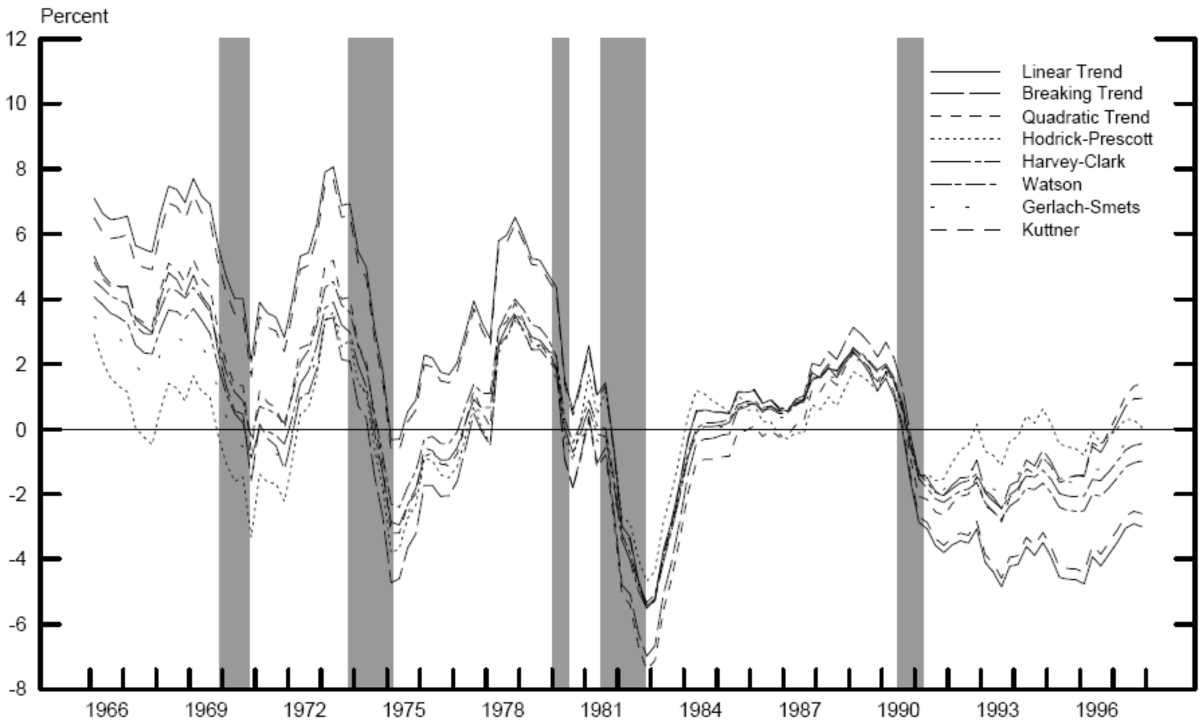
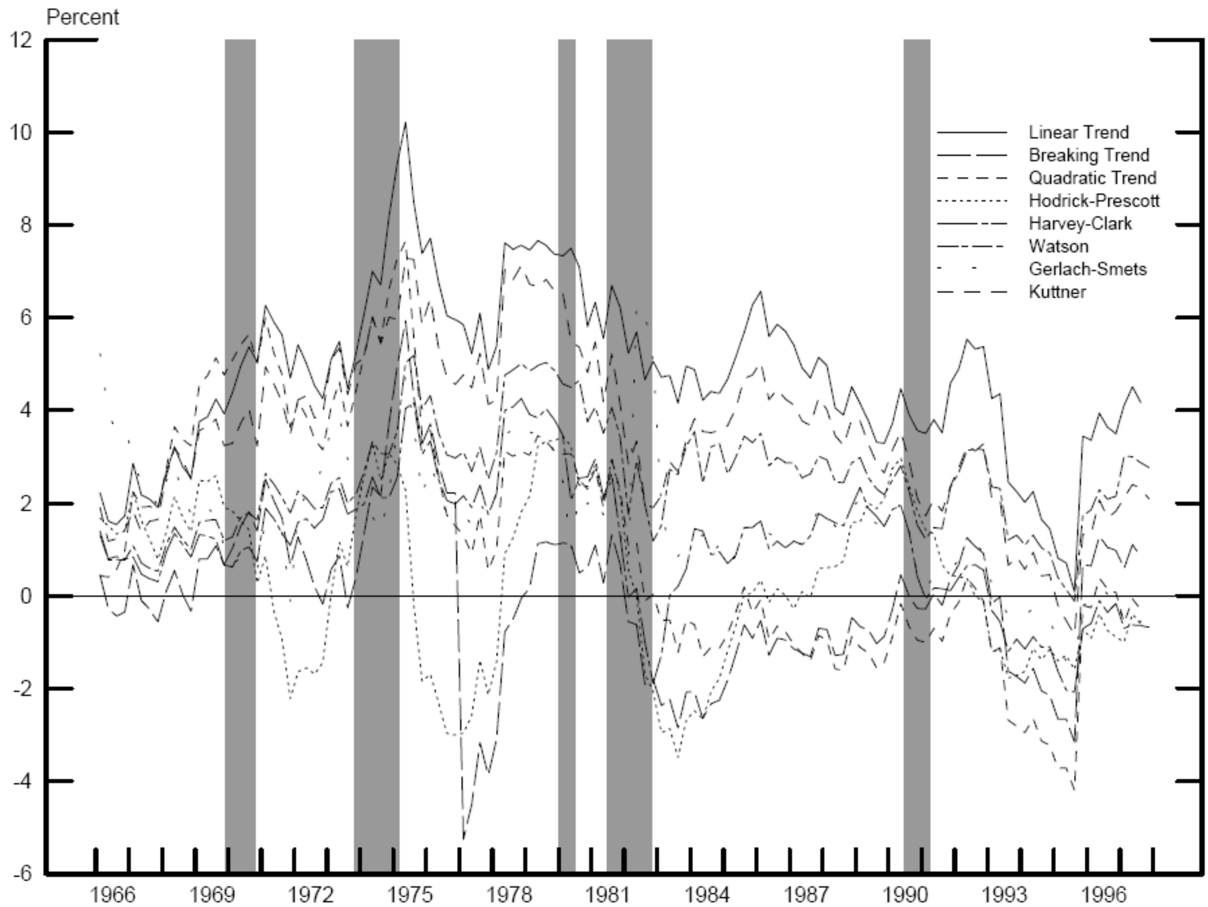


Figure 1 shows real-time and final estimates of the output gap. The shaded bars indicate recessions as dated by the National Bureau of Economic Research. All detrending methods seem to follow the same general movement. Nevertheless the methods give rise to quite different estimates of the level of the output gap (consider 1966-1969 and 1982-1990). The final estimates lie closer together than the real-time estimates. Comparing the two graphs, the revisions – depicted in figure 2 – seem rather large.

Figure 2

Total Revision in Business Cycle Estimates



OvN note that the revisions are often of the size of the output gap itself! "All revisions are highly persistent, with coefficients ranging from 0.8 for the Gerlach-Smets model to 0.96 for the quadratic trend."
Summary statistics of all detrending methods can be found in table 1.

TABLE 1.—OUTPUT-GAP SUMMARY STATISTICS

Method	*MEAN*	*SD*	*MIN*	*MAX*	*COR*
Hodrick-Prescott					
Final	0.04	1.65	−4.67	3.60	1.00
Quasi-real	−0.12	1.70	−3.96	3.79	0.55
Real-time	−0.27	1.90	−6.63	3.84	0.49
Breaking trend					
Final	0.18	2.58	−6.98	5.31	1.00
Quasi-real	0.56	2.79	−6.55	7.02	0.85
Real-time	0.21	3.15	−10.52	5.02	0.82
Quadratic trend					
Final	0.30	2.72	−7.39	5.20	1.00
Quasi-real	−0.70	2.71	−7.23	6.19	0.60
Real-time	−0.96	3.03	−10.83	4.70	0.58
Linear trend					
Final	1.30	3.87	−5.44	8.06	1.00
Quasi-real	−2.65	3.49	−10.32	7.02	0.88
Real-time	−3.45	3.98	−10.52	5.02	0.89
Watson					
Final	0.45	2.37	−5.34	4.56	1.00
Quasi-final	−0.26	2.19	−5.07	5.06	0.95
Quasi-real	−1.71	2.37	−7.31	4.42	0.83
Real-time	−2.08	2.61	−7.43	3.56	0.89
Kuttner					
Final	1.20	3.63	−5.52	7.69	1.00
Quasi-final	0.78	3.51	−5.61	6.92	0.99
Quasi-real	−1.63	2.79	−6.81	6.23	0.87
Real-time	−2.37	3.16	−7.91	4.86	0.88
Harvey-Clark					
Final	0.25	2.17	−5.51	4.06	1.00
Quasi-final	−0.71	1.53	−4.62	3.21	0.89
Quasi-real	−0.66	1.60	−4.14	3.41	0.81
Real-time	−0.93	1.91	−6.99	3.02	0.77
Gerlach-Smets					
Final	0.08	1.95	−5.37	3.51	1.00
Quasi-final	−0.57	1.55	−4.85	3.30	0.92
Quasi-real	−0.89	2.57	−13.17	1.95	0.56
Real-time	−1.57	2.08	−11.05	0.90	0.75

The alternative detrending methods are as described in the text. The statistics shown for each variable are: *MEAN*, the mean; *SD*, the standard deviation; and *MIN* and *MAX*, the minimum and maximum values. *COR* denotes the correlation with the final estimate of the gap for that method. All statistics are for 1966:1–1997:4.

Table 3 evaluates the revisions for each detrending method. Note the last column OPSIGN which indicates the frequency the real-time estimates provided the opposite sign of the final estimates. The breaking trend seems to perform best with only 22% as opposed to the Kuttner and linear trend method which perform worst with almost 50% wrong signs. This and the average of 40% of wrong signs for all methods shows that "the errors associated with real time-estimates of the output gap are substantial."

Table 3

Summary Reliability Indicators

Method	COR	NS	NSR	OPSIGN
Hodrick-Prescott	0.49	1.10	1.11	0.41
Breaking Trend	0.82	0.69	0.69	0.22
Quadratic Trend	0.58	0.97	1.07	0.35
Linear Trend	0.89	0.47	1.32	0.49
Watson	0.89	0.49	1.17	0.42
Kuttner	0.88	0.48	1.09	0.49
Harvey-Clark	0.77	0.64	0.84	0.34
Gerlach-Smets	0.75	0.73	1.11	0.41

Notes: The table shows measures evaluating the size, sign and variability of the revisions for alternative methods. COR, denotes the correlation of the real-time and final estimates (from Table 1). NS indicates the ratio of the standard deviation of the revision and the standard deviation of the final estimate of the gap. NSR indicates the ratio of the root mean square of the revision and the standard deviation of the final estimate of the gap. OPSIGN indicates the frequency with which the real-time and final gap estimates have opposite signs.

Figure 3

Estimated Business Cycle: Linear Trend

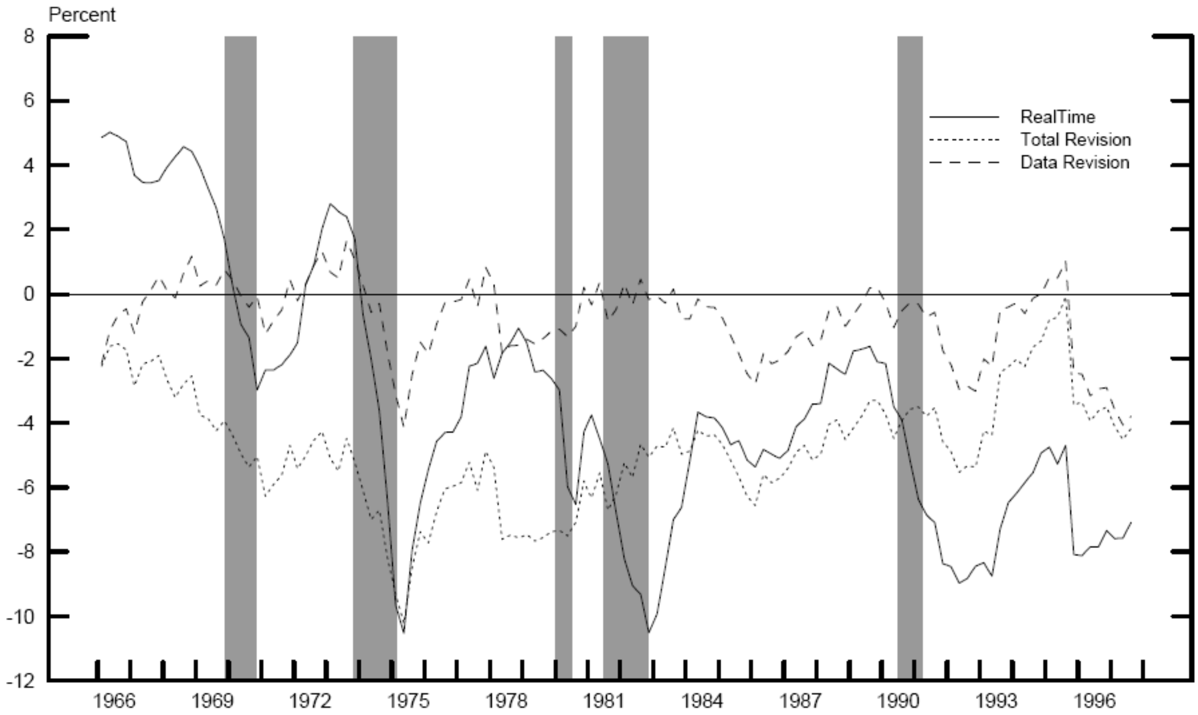


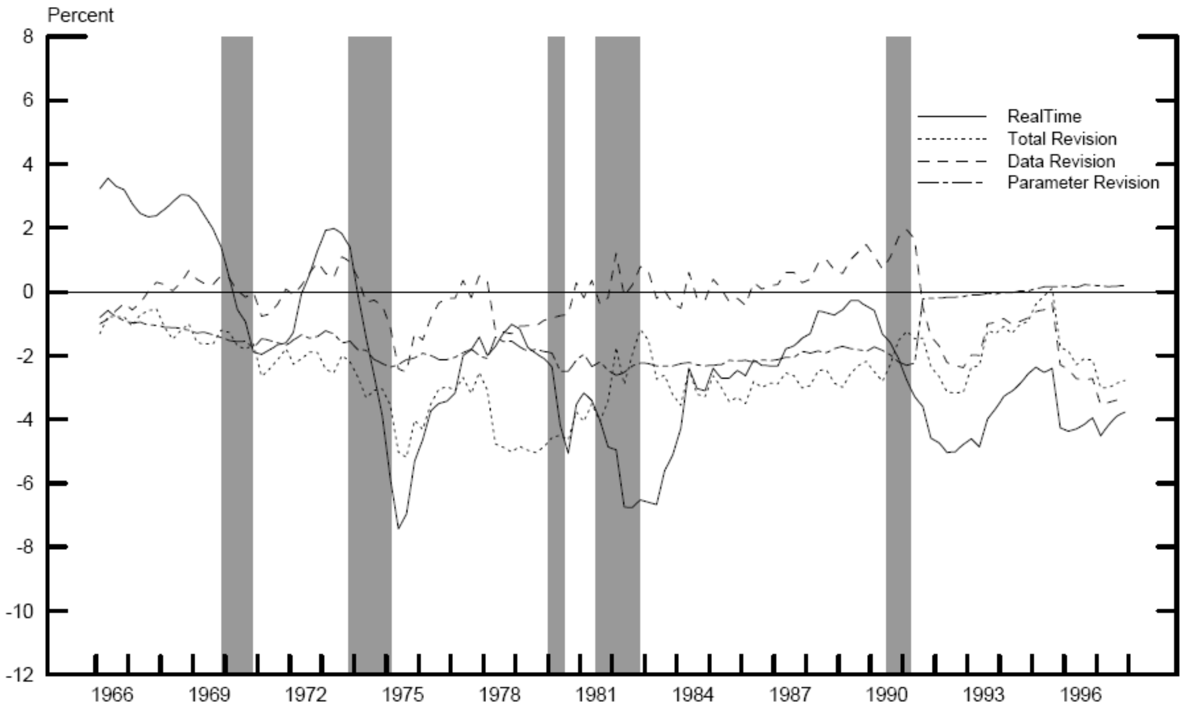
Figure 3 shows the decomposition of the revision of the linear trend method. The total revision is often of the same size as the real-time estimate. Looking at the trough in 1975 this means that whereas the real-time estimates clearly implied being in a recession, the final estimates indicated a closed output gap (0%)!

We can observe for all detrending methods that data revision (=real-time minus quasi real estimates) only plays a minor role in explaining total revision. Availability of new data seems to be the biggest source of revision.

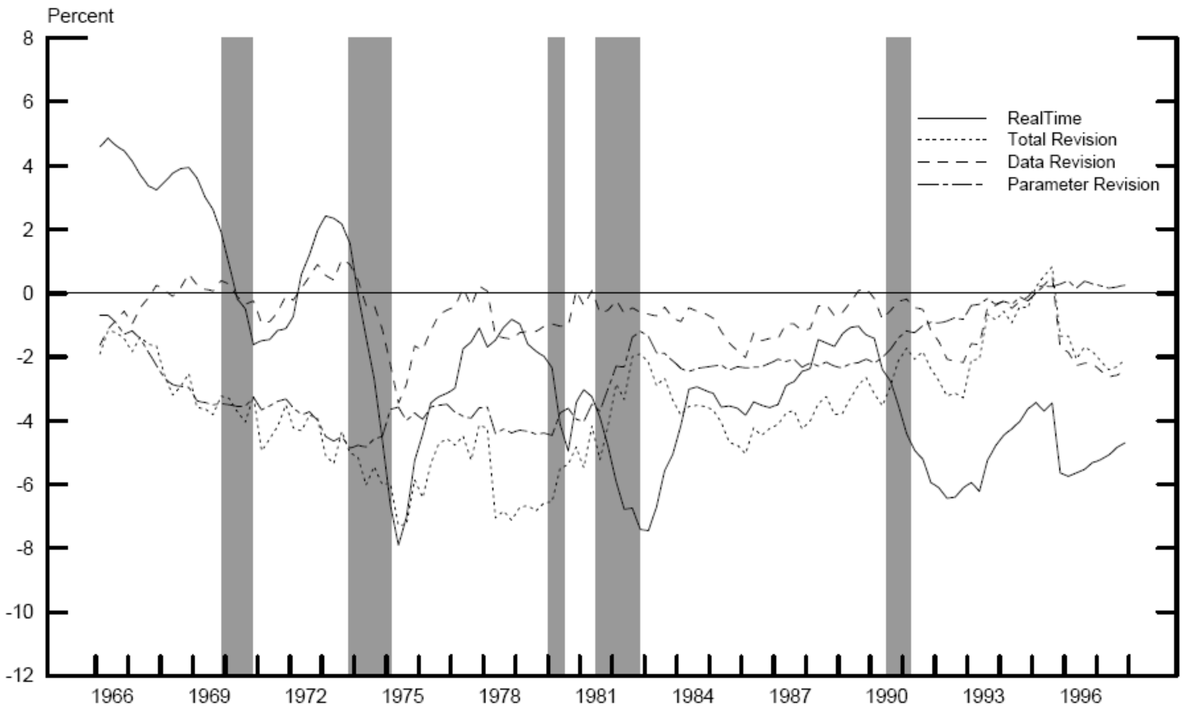
Figure 7 depicts the output gap estimates and revisions of the univariate unobserved components model by Watson and its bivariate counterpart, the Kuttner model. Both graphs indicate that the effect of data revision was negligible, whereas the parameter revisions seem to have caused most of the total revision. We again find that the revision is of the same amplitude as the output gap itself. The additional information contained in the Phillips curve does not significantly improve the output gap estimate.

Figure 7

Estimated Business Cycle: Watson



Estimated Business Cycle: Kuttner

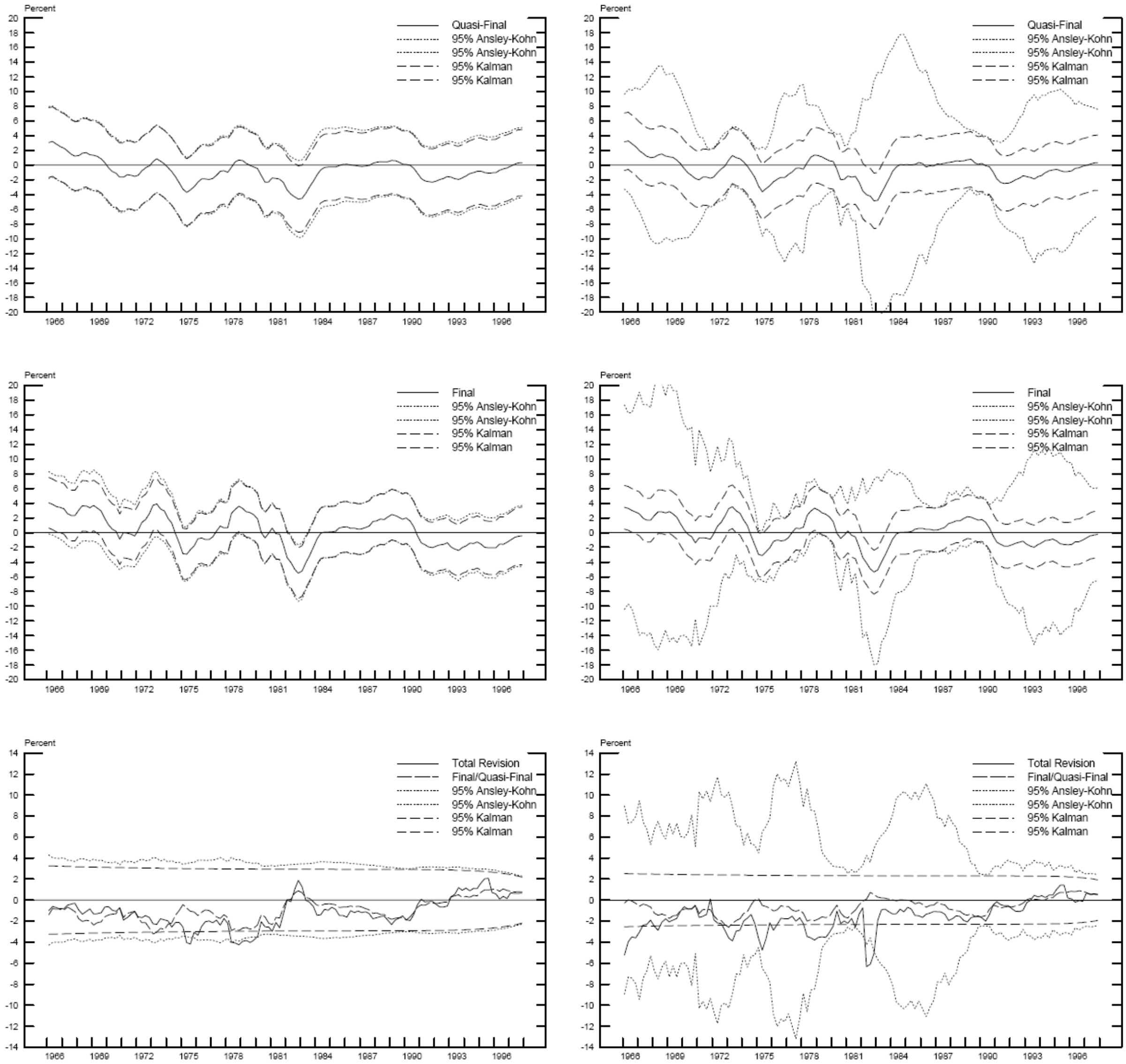


Finally we turn to the confidence intervals: Figure 9 shows the 95% Kalman and Ansley-Kohn confidence intervals for the quasi final and for the final estimates, as well as total revision together with final-quasi final revision (from top to bottom).

Figure 9

Estimates and 95% Confidence Intervals

Harvey-Clark **Gerlach-Smets**



Where as the Kalman bands are narrower for the Gerlach-Smets model, its Ansley-Kohn bands tend to be wider. “[...] in the absence of parameter uncertainty, incorporating information from the Phillips curve based on the final data helps narrow the uncertainty of the estimated output gaps. However, this narrowing is reversed when parameter uncertainty is taken into account.” What seems most shocking is that for both models, the estimated output gap is never significantly different from zero! The bottom graphs indicate that revisions do not seem to be unusually large relative to their confidence intervals.

3.4) Conclusions

"[...] The reliability of output gap estimates in real time tends to be quite low. The revisions are of the same order of magnitude as the estimated output gap itself for all the methods examined. [...] For UC [unobserved components] models, we find that multivariate methods that incorporate information form inflation to estimate the output gap are not more reliable than their univariate counterparts. [...] The revision of published data does not appear to be the primary source of revisions for the methods we examined. Rather, the bulk of the problem is due to the pervasive unreliability of end-of-sample estimates of the output trend.
[...] In the light of the unreliability of real-time estimates of the output gap, great caution is required in their use."